**Lesson Plan 2**

A.P. Calculus AB/Calculus I

Unit II/VII: Limits & Continuity

Segment iii/iv: Relationship Between Continuity & Limits

Lesson 2/4: Classes of Discontinuous Functions

Purpose and Central Focus

The purpose of this lesson is to construct a taxonomy of discontinuous function. While classifying discontinuities is a worthy mathematical activity in its own right, it also provides new insights into continuity, as the types of discontinuous function embody different things that can “go wrong” with continuity. Further, some of the instances in which continuity is a hypothesis later can be relaxed using the classification – for example, discontinuous functions up to class 2 are Riemann integrable in a natural way, while those of class 1 can have a derivative defined sensibly.

This lesson is the second in a learning segment on the relationship between limits and continuity. The understandings of continuity built in the first lesson will be here put to use and their limitations probed. This will further use and build students’ skills at precise communication about these concepts.

Illinois Learning Standards/Common Core Standards/Content Area Standards

* A.P. Calculus LO 1.2A: Analyze functions for intervals of continuity or points of discontinuity.
* A.P. Calculus LO 1.1D: Deduce and interpret behavior of functions using limits.
* IL SLE 2C-J 7. Demonstrate strategies for collaborating with peers, adults, and others in the community.

Objectives

1. Given a symbolic or graphic representation of a discontinuity in a function (C), the student (A) will accurately classify the discontinuity (B) in 90% of cases (D).
2. During small group work to produce arguments for and against possible relations between continuity and limit behavior (C), the student (A) will verbally share candidate ideas with the group (B) at least twice per period (D).

Duration

45 minutes

Materials and Equipment

* Presentation for introductory discussion [can be adapted for manual presentation.]
* Means to present introductory discussion and student work, probably computer with projector [if using manual presentation, black- or whiteboards, computer with document camera, or overhead projector with transparencies will do.]
* Several copies of cards and worksheet for group card sort activity.
* Pre-existing groups or means to quickly make groups, perhaps grouping cards.
* Homework assignment, or prepared list of appropriate problems from textbook [or online homework system (OHS) and OHS version of assignment.]
* Student and teacher textbooks. (Designed for CPM Calculus, 2nd Ed.)

Gaining Attention/Introduction/Anticipatory Set (2 minutes)

In order to motivate students, we want to present them with a challenge that might surprise and challenge their understanding. Consequently, we present a graph of a function with a finite jump discontinuity at *x* = 0 and challenge students to deduce the limit of the function as it approaches this point. The denouement of this challenge should not be dealt with now, but rather toward the end of the “Stimulate Recall” step – however, it should be left displayed until dealt with.

Inform Learners of Objectives (3 minutes)

The objective of this lesson flows very naturally from the question posed naturally in our previous lesson: “Are all discontinuities more or less the same?” Re-pose this question, soliciting a couple quick answers. Then, orally deliver the objective in the language of that previous question: “Today, our objective is to look at the details of how discontinuities can be different and the same. To do that, we’ll compare and contrast many discontinuous functions. Our ideas about continuity – which, again, is a central idea for calculus – may be extended or refined by doing this.” We write down on a sideboard or otherwise display throughout the lesson a short form of this goal, along the lines of: “Classify the different possible sorts of discontinuity.”

Stimulate Recall of Prior Learning (5 minutes)

The class should review selected problems from previous homework, with students demonstrating and explaining their results related to continuity. We provoke questions from other students throughout this, such as “Who can relate this to a (formal/informal) notion of continuity?” or “Did we assume anything here?”

Summarize these continuity results with special emphasis on: (i) points of undeveloped understanding based on previous lesson’s diagnostic and results and (ii) formal definition of continuity introduced in the previous lesson. A crucial note to deliver is: “Continuity at a point is an ‘all-or-nothing deal’ – if any of the things we need isn’t there, the function can’t be continuous.” This recapitulates the close reading of the definition in the previous lesson, as well as the digested, ramified version of the definition in the text.

Present the Content (10 minutes)

We now return to the challenge we presented at the start of class. “What is the limit of this function at the indicated point?” Solicit some student answers; discuss these, engaging formal and graphical understanding of limits from earlier in the unit; and (if students haven’t hit on it) reveal the punch line: there is no limit – it simply doesn’t exist.

Perhaps our tool has an inadequacy here. Discuss this, using questions like “What went wrong with the limit here?” “Does this invalidate the idea of limits?” and “Is there any more refined tool we can use to say something about the function at that point?” If students don’t mention handed limits (covered in a previous segment), remind them, noting that the function has a right- and left-hand limit at the point of discontinuity, but these are not equal.

To wrap this section, challenge students to connect this to our objective. Ask: “Is this function continuous?” On consensus that it’s not, ask: “What went wrong – where did it fail to fit the definition?” Students should reach the answer that it fails to have a limit at the point at all, so it can’t be equal to its limit as the definition requires. Students may further add, after our questioning or organically, that the function’s handed limits exist and that it is defined or undefined at the point, depending on the exact function we drew. Close with a preview: “Get in groups and let’s consider some other functions in a similar light.”

Provide Learning Guidance (15 minutes)

Launch the card sort activity. Students transition to heterogeneous groups as cards and worksheets are distributed. In the activity, groups of students discuss and create clusters of functions based on mathematical characteristics. Throughout the activity, circulate, noticing groups’ progress, the quality of their explanations, and discrepancies from other groups to sequence the presentations at the end of the activity and in order to target assessing and advancing questions (examples below). Notice contributions of members to group work.

It is most important to notice groups’ actions right toward the start of the activity, to ensure that they begin, and then at a natural checkpoint when they classify all the functions with jump discontinuities, similar to our example – expect most groups to sort those out first, but some to initially stop at that point, leaving classes of jump discontinuous and “other.”

Assessing questions we might use include:

* “What categories have you come up with so far?”
* “What rules govern the categories/What makes these functions the same as those/different from those?”
* “How is this related to our formal definition of continuity?”
* “What would you say to someone who said this function belongs in that class?”
* “Can you explain what [other group member] just said?”

Given the somewhat arbitrary nature of this classification, it will be highly useful to have a stock of advancing questions, such as:

* “Do your categories cover every function you have a card for? What about any sorts of functions you don’t have a card for/could make up?”
* “Does each function fit in just one class?”
* “Are all these functions really alike/different?”
* “Are there any differences you can spot between some of the functions in this category?”

Elicit Performance (Practice) (0 minutes – outside class)

Students do a homework assignment [perhaps in OHS] in which they use the system our class has derived to categorize various functions, justifying their choice in each case. Student work is graded and returned for formative assessment.

Provide Feedback (8 minutes)

Teacher questions during group work period provides first line of feedback, though this is indirect and at the group level. Main feedback will occur during presentations of schemes and homework grades (which will be given later).

When 10 minutes remain, or when most groups have finished their classification schemes, students present their schemes for questioning as we sequence. We may choose to have a group present their whole scheme, or only aspects of it that are interesting or different from others. Ideally, about 3 groups will present, so our knowledge of the state of schemes vis-à-vis others will be important to make sure that everyone’s view gets covered.

Ideally, students will interrogate or note differences in the schemes presented, resulting in dialogue with the presenters. However, to further provoke or even create this, it will be useful for us to have a number of questions ready, such as:

* “I noticed you had a different result. What was the difference?” [to a group known to have a different classification for one or more functions].
* “Who can connect this result to the formal definition of continuity?”
* “Can you re-state their classification criterion in a (more/less) formal way?”
* “Does this scheme cover all the functions? Does it put each in only one place?”
* “Is this the only valid way to do it?”

The focus of presentations is reasoning for classification and persuasion that the scheme is good – that it covers all functions, puts each in only one category, and is closely related to the definition of continuity. It is important that students leave this phase with these common errors corrected:

* Use of only informal criteria and/or non-relation of scheme to formal continuity definition.
* Incomplete classification – e.g. “Jump discontinuous” and everything else.
* Misclassification.
* Lapses in precision of language.
  + Note that groups may come up with their own names for classes of discontinuity – this is acceptable and even encouraged. It will be important to provide them a translation to the more generally accepted names, however, as will be done later.

Students should spot and correct errors through dialogue, especially if sequencing and talk moves are successful. However, direct guidance may be necessary, especially if misunderstandings persist toward the end of the presentations. As students near the end of presentations, correct any essential errors and assemble a table of the class’ categories for later display.

Each group’s scheme is collected for formative assessment.

Assess Performance (0 minutes – in parallel with other activities and outside class)

For the first objective, responses to assessing questions provide informal formative assessment at the group level. Student contributions to presentations, especially, provide good evidence along these lines. The group’s categorization scheme provides further information, albeit aggregated and for the converse goal. A more direct and individual formative assessment is available by considering student results on homework. Summative assessment is done by a segment quiz.

Achievement of the second, social objective is only assessed in an informal formative fashion in this lesson. Student responses during group work provide a direct measure, while contributions during various discussion/question-and-answer portions provide a proxy. Summative assessment can be done by charting responses in later lesson involving bona fide group work. Alternatively, summative assessment can be done on this lesson by charting some groups or members, if a longer baseline or segmentation of assessment are desired.

Enhance Retention and Transfer (2 minutes)

Wrap lesson by directly covering the following points:

* Show the classes of discontinuity table [project or perhaps write on board] and have all students copy this down. If necessary, provide a translation between the class’s names for the categories and the text’s – remind students to use formal names on homework, but translating between our personal names and formal names is fine.
* Re-state the formal definition of continuity once again. Correlate the categories to the hypotheses in the definition.
* Leave students with a modeling question: “Ponder this for tomorrow: What do these different classes mean in the real world? Is there anything real that looks like, say, a jump discontinuous function or one with vertical asymptotes?”