MR. SORICE'S CALCULUS I
SUMMATIVE ASSESSMENT FOR SEGMENT

NAME: ________________________________

Quiz Directions

This quiz is a chance to show some of what you've learned about limits and continuity. There are 62 points available.

This quiz has several sections, each of which may have its own directions. Read those carefully before you begin to answer.

Please continue to follow all our usual test expectations, especially those for when you have finished.

Stand-alone calculators are allowed.

Good luck!

True/False – 1 point each

Each statement is either True or False. Circle the correct value. There is no penalty for guessing.

Question 1. A function can have an essential discontinuity on an interval, but still be defined everywhere on that interval.

- True
- False

Question 2. If a function is defined everywhere on an interval, then it's continuous everywhere on that interval.

- True
- False

Question 3. It's possible that $\lim_{x\to a} f(x)$ does not exist, even though both $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ do.

- True
- False

Question 4. Suppose $f(x)$ is discontinuous at $x = 0$. Nevertheless, f may still take on a finite limit as $x \to 0$.

- True
- False

Short Answer (Written) – 4 points each

Respond to each question concisely in 1-2 complete sentences.

Question 5. Let

$$f(x) = \begin{cases} x, & x < 0 \\ 0, & x = 0 \\ x^3, & x > 0 \end{cases}.$$

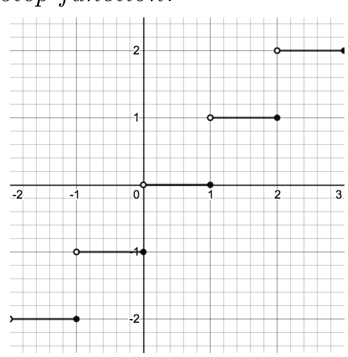
Classify the continuity behavior of f at $x = 0$.

Question 6. We know the following about $f(x)$:

- It has a point discontinuity at $x = 2$.
- However, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$.

What must be true about $f(2)$ and why?

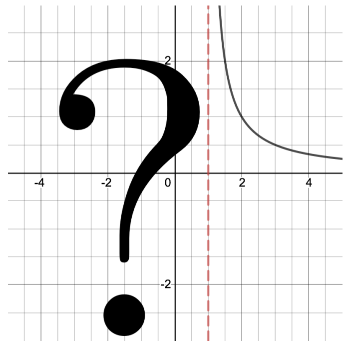
Question 7. This is a *step function*:



It's flat almost everywhere, but it goes up 1 unit each time x goes right 1 unit.

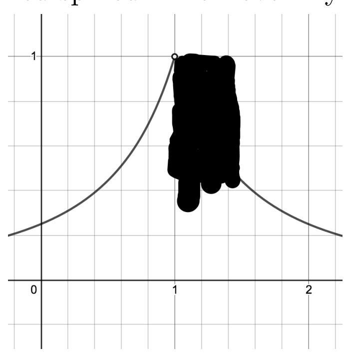
Which class of discontinuity does the step function exhibit at every integer value of x we can see? In terms of limit behavior, how do we know that?

Question 8. We know this function is defined for $x < 1$, but we can't see what it looks like there:



What kind of discontinuity or discontinuities can it have at $x = 1$? Why?

Question 9. Oh man, Ned spilled ink all over my graphs!



I'm sure the right branch is under that blot, but I don't remember where. Agnes says the function had a removable discontinuity.

What could the right branch do if that's true?

Question 10. My friend Mona says she thinks the function from Question 9 was continuous everywhere.

What might you say to settle this disagreement?

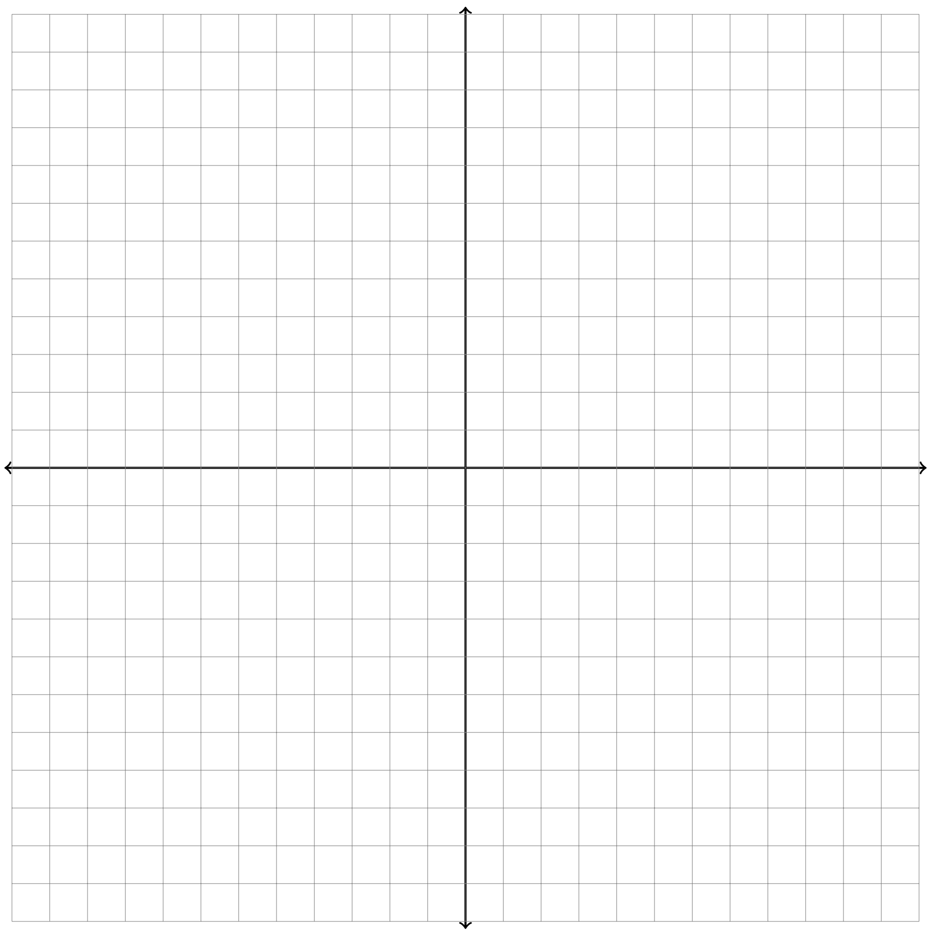
Short Answer (Graphs) – 4 points each

For each question, use the provided graph paper to draw 1 or 2 graphs of functions that exhibit the specified continuity or limit behavior.

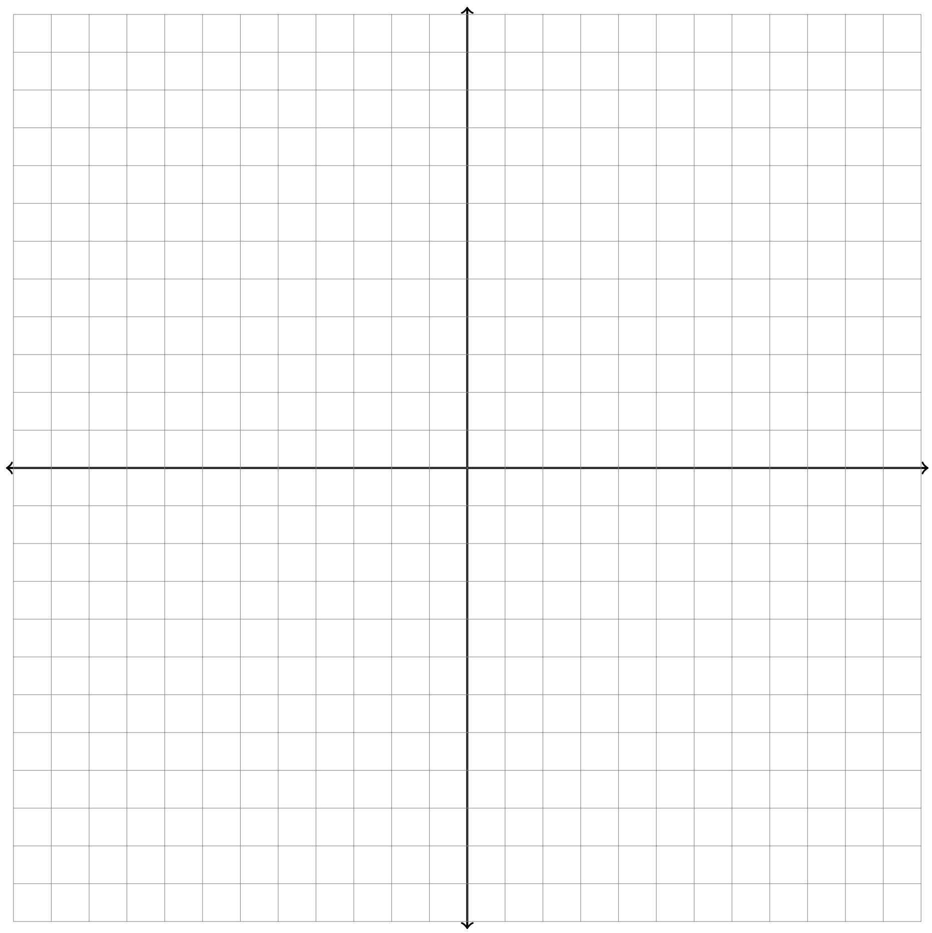
Please label vertical asymptotes.

Beware: you will lose credit on a problem if any of your drawings contradicts the given behavior, or is of a non-function.

Question 11. A function with an essential discontinuity at $x = 2$, a jump discontinuity at $x = 5$, and that is undefined but takes on a limit at $x = 0$.



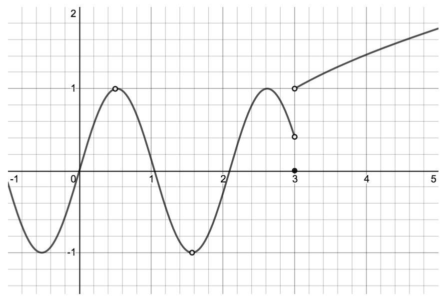
Question 12. A function with different left- and right-sided limits at $x = -3$, takes on a limit other than its value at $x = 0$, and becomes infinite somewhere in its domain.



Multiple Choice – 2 points each

Each question has a single correct answer. Circle the letter corresponding to it. There is no penalty for guessing.

The next 4 questions are about this function:



Question 13. What sort of discontinuity is present at $x = 3$?

a) No discontinuity – it's continuous
b) Removable discontinuity
c) Jump discontinuity
d) Essential discontinuity
e) We can't say

Question 14. What is the function's behavior at $x = \frac{1}{2}$?

a) There's no discontinuity – it's continuous
b) It has a removable discontinuity
c) It has a jump discontinuity
d) It has an essential discontinuity
e) We can't say

Question 15. Classify the function's behavior as $x \to 0^-$?

a) There's no discontinuity – it's continuous
b) It has a removable discontinuity
c) It has a jump discontinuity
d) It has an essential discontinuity
e) We can't say

Question 16. How many discontinuities are there? (Assume there are none outside the graph window.)

a) 2
b) 3
c) 4
d) 5
e) We can't say

In the next 3 questions, consider this function, which depends on the constants m and b:

$$f(x) = \begin{cases} 10^x, & x < 0 \\ 0, & x = 0 \\ mx + b, & x > 0 \end{cases}.$$

Question 17. If $m = 0$ and $b = 5$, what sort of discontinuity is there at $x = 0$?

a) No discontinuity – it's continuous
b) Removable discontinuity
c) Jump discontinuity
d) Essential discontinuity
e) We can't say

Question 18. Which combination of m and b makes f continuous at $x = 0$?

a) $m = 0, b = 0$
b) $m = 0, b = 1$
c) $m = 1, b = 0$
d) $m = 1, b = 10$
e) None of these.

Question 19. Which combination of m and b creates a function that takes on a defined limit as $x \to 0$?

a) $m = 0, b = 0$
b) $m = 0, b = 1$
c) $m = 1, b = 0$
d) $m = 1, b = 10$
e) None of these.

Essay – 12 points

Georg has an interesting idea. He thinks if he knows the limit behavior of a function at all points in an interval, he'll know everything about the function.

Write an essay of 2-3 paragraphs critiquing Georg's idea. In your discussion, consider *left- and right-handed limits* and *classes of discontinuous functions*. Conclude your essay with a claim about the soundness of Georg's idea. If you conclude that it works as is, share any limitations you see on it; on the other hand, if you conclude that the idea is lacking, discuss ways it might be repaired.

Feel free (and encouraged!) to explain your ideas using various representations of functions, including symbols and graphs, as well as other mathematical language. Incorporate these with your usual academic writing style.