Study Guide and Intervention

Solving Polynomial Equations

Factor Polynomials

Techniques for Factoring Polynomials	For any number of terms, check for: greatest common factor
	For two terms, check for: Difference of two squares $a^2 - b^2 = (a + b)(a - b)$ Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
	For three terms, check for: Perfect square trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ General trinomials $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
	For four or more terms, check for: Grouping $ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Example **Factor $24x^2 - 42x - 45$.**

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the x terms, you must find two numbers whose product is $8 \cdot (-15) = -120$ and whose sum is -14. The two coefficients must be -20 and 6. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$ Group to find a GCF.

$= 4x(2x - 5) + 3(2x - 5)$ Factor the GCF of each binomial.

$= (4x + 3)(2x - 5)$ Distributive Property

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14x^2y^2 + 42xy^3$ **2.** $6mn + 18m - n - 3$ **3.** $2x^2 + 18x + 16$

4. $x^4 - 1$ **5.** $35x^3y^4 - 60x^4y$ **6.** $2r^3 + 250$

7. $100m^8 - 9$ **8.** $x^2 + x + 1$ **9.** $c^4 + c^3 - c^2 - c$

5-5 Study Guide and Intervention *(continued)*

Solving Polynomial Equations

Solve Polynomial Equations If a polynomial expression can be written in quadratic form, then you can use what you know about solving quadratic equations to solve the related polynomial equation.

Example 1 **Solve $x^4 - 40x^2 + 144 = 0$.**

$x^4 - 40x^2 + 144 = 0$			Original equation
$(x^2)^2 - 40(x^2) + 144 = 0$			Write the expression on the left in quadratic form.
$(x^2 - 4)(x^2 - 36) = 0$			Factor.
$x^2 - 4 = 0$	or	$x^2 - 36 = 0$	Zero Product Property
$(x - 2)(x + 2) = 0$	or	$(x - 6)(x + 6) = 0$	Factor.
$x - 2 = 0$ or $x + 2 = 0$	or	$x - 6 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 2$ or $x = -2$	or	$x = 6$ or $x = -6$	Simplify.

The solutions are ± 2 and ± 6.

Example 2 **Solve $2x + \sqrt{x} - 15 = 0$.**

$2x + \sqrt{x} - 15 = 0$	Original equation
$2(\sqrt{x})^2 + \sqrt{x} - 15 = 0$	Write the expression on the left in quadratic form.
$(2\sqrt{x} - 5)(\sqrt{x} + 3) = 0$	Factor.
$2\sqrt{x} - 5 = 0$ or $\sqrt{x} + 3 = 0$	Zero Product Property
$\sqrt{x} = \frac{5}{2}$ or $\sqrt{x} = -3$	Simplify.

Since the principal square root of a number cannot be negative, $\sqrt{x} = -3$ has no solution.

The solution is $\frac{25}{4}$ or $6\frac{1}{4}$.

Exercises

Solve each equation.

1. $x^4 = 49$

2. $x^4 - 6x^2 = -8$

3. $x^4 - 3x^2 = 54$

4. $3t^6 - 48t^2 = 0$

5. $m^6 - 16m^3 + 64 = 0$

6. $y^4 - 5y^2 + 4 = 0$

7. $x^4 - 29x^2 + 100 = 0$

8. $4x^4 - 73x^2 + 144 = 0$

9. $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$

10. $x - 5\sqrt{x} + 6 = 0$

11. $x - 10\sqrt{x} + 21 = 0$

12. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$