NAME ______________________ DATE ____________ PERIOD ____________

Study Guide and Intervention

Operations with Polynomials

Multiply and Divide Monomials **Negative exponents** are a way of expressing the multiplicative inverse of a number.

Negative Exponents	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer n.

When you **simplify an expression**, you rewrite it without powers of powers, parentheses, or negative exponents. Each base appears only once, and all fractions are in simplest form. The following properties are useful when simplifying expressions.

Product of Powers	$a^m \cdot a^n = a^{m+n}$ for any real number a and integers m and n.
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers m and n.
Properties of Powers	For a, b real numbers and m, n integers: $(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0, b \neq 0$

Example **Simplify. Assume that no variable equals 0.**

a. $(3m^4n^{-2})(-5mn)^2$

$$\begin{aligned}(3m^4n^{-2})(-5mn)^2 &= 3m^4n^{-2} \cdot 25m^2n^2\\ &= 75m^4m^2n^{-2}n^2\\ &= 75m^{4+2}n^{-2+2}\\ &= 75m^6\end{aligned}$$

b. $\dfrac{(-m^4)^3}{(2m^2)^{-2}}$

$$\begin{aligned}\frac{(-m^4)^3}{(2m^2)^{-2}} &= \frac{-m^{12}}{\frac{1}{4m^4}}\\ &= -m^{12} \cdot 4m^4\\ &= -4m^{16}\end{aligned}$$

Exercises

Simplify. Assume that no variable equals 0.

1. $c^{12} \cdot c^{-4} \cdot c^6$

2. $\frac{b^8}{b^2}$

3. $(a^4)^5$

4. $\frac{x^{-2}y}{x^4y^{-1}}$

5. $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$

6. $\left(\frac{x^2y}{xy^3}\right)^2$

7. $\frac{1}{2}(-5a^2b^3)^2(abc)^2$

8. $m^7 \cdot m^8$

9. $\frac{8m^3n^2}{4mn^3}$

10. $\frac{2^3c^4t^2}{2^2c^4t^2}$

11. $4j(-j^{-2}k^2)(3j^3k^{-7})$

12. $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$

Study Guide and Intervention *(continued)*

Operations with Polynomials

Operations with Polynomials

Polynomial	a monomial or a sum of monomials
Like Terms	terms that have the same variable(s) raised to the same power(s)

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1 **Simplify $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$.**

$4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$

$= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y$ Distribute the minus sign.

$= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy)$ Group like terms.

$= x^2y - xy^2 - 8xy$ Combine like terms.

You use the distributive property when you multiply polynomials. When multiplying binomials, the **FOIL** pattern is helpful.

FOIL Pattern	To multiply two binomials, add the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

Example 2 **Find $(6x - 5)(2x + 1)$.**

$(6x - 5)(2x + 1) = 6x \cdot 2x + 6x \cdot 1 + (-5) \cdot 2x + (-5) \cdot 1$

First terms, Outer terms, Inner terms, Last terms

$= 12x^2 + 6x - 10x - 5$ Multiply monomials.

$= 12x^2 - 4x - 5$ Add like terms.

Exercises

Simplify.

1. $(6x^2 - 3x + 2) - (4x^2 + x - 3)$

2. $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$

3. $(-4m^2 - 6m) - (6m + 4m^2)$

4. $27x^2 - 5y^2 + 12y^2 - 14x^2$

5. $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$

6. $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$

Find each product.

7. $2x(3x^2 - 5)$

8. $7a(6 - 2a - a^2)$

9. $(x^2 - 2)(x^2 - 5)$

10. $(x + 1)(2x^2 - 3x + 1)$

11. $(2n^2 - 3)(n^2 + 5n - 1)$

12. $(x - 1)(x^2 - 3x + 4)$