**Lesson Plan 3**

4-8: Rational Zero Theorem

Purpose and Central Focus

This lesson is part of a learning segment on the roots of polynomials. The purpose of this lesson is to teach a new technique for finding roots of polynomials over the integers, the Rational Zero Theorem (R.Z.T.). This is an exhaustive technique that combines elegantly with tools developed in previous lessons to create a formidable arsenal for attacking problems involving polynomials. This sequence is designed to create young mathematicians who can proficiently and flexibly use these techniques in combination.

An additional shade of interest in the R.Z.T. is in its underlying mathematical logic. It is an important case in which converses very much do not hold: it produces a list such that, if there are rational zeros, they are on the list – however, neither tempting converse holds: there may be other zeros (irrational and/or non-real) that are not on the list, and not all items on the list are zeros! In using this technique in combination with others, then, students are challenged to refine their conceptual understanding, increase their problem-solving ability, and become fluent users of procedures.

Illinois Learning Standards/Common Core Standards/Content Area Standards

* HSA.APR.B.3 – Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
* HSA.SSE.A.2 – Use the structure of an expression to identify ways to rewrite it. For example, see *x*4 – *y*4 as (*x*2)2 – (*y*4)2, thus recognizing it as a difference of squares that can be factored as (*x*2 – *y*2)(*x*2 + *y*2).

Objectives

1. Given a polynomial with integer coefficients *P* (C), the learner (A) will use the Rational Zero Theorem to generate the list of candidate rational zeros (B), missing at most 2 (including sign changes) (D).
2. Given a polynomial *P* and a candidate zero *c* (C), the learner (A) will use various techniques to efficiently determine whether *c* is a zero of *P* AND, if it is, to generate the depressed polynomial *P*(*x*) ÷ (*x* – *c*) (B), in 80% of cases (D).

Duration

47 minutes.

Materials and Equipment

* Student note packet.
* Video tutorials for direct presentation and example exercises.
  + Website to disseminate of these.
* Student and teacher textbooks (McGraw Hill *Integrated Math 3*).
  + Student answer sheets for marking previous work (**4-7: odd exercises 17-25, 33-43, 47-49 & 61-71**).
  + Worked out pre-assessment exercises (**4-8: odd exercises 1-9**).
* Graphing calculators.

Stimulate Recall of Prior Learning I (3 minutes)

Students mark their own results on classwork from previous lesson (**4-7: odd exercises 17-25, 33-43, 47-49 & 61-71**). Students may ask for review of certain exercises, but it is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Gaining Attention/Introduction/Anticipatory Set (3 minutes)

Class uses flipped structure, so students come to class having viewed video tutorials and done pre-assessment exercises. After checking their note work, I make the following claim for students to respond to: “Finding zeros is easy. I just plug in numbers until I find one, then repeat.”

Inform Learners of Objectives (2 minutes)

After hearing a couple student responses, I reinforce or broach the fact that there are an infinite number of numbers to check, and we might not always be lucky enough to find a root, even if we check for a long time or limit ourselves to integers or rational numbers. This is where the R.Z.T. does its work. It gives us a list and, if we check everything on that list, we’re guaranteed to find all the rational zeros (if there are any). Once we’ve checked all those, we know we should use a different strategy than guess and evaluate. It also saves us from guessing things that have no chance to be a root.

Then I state our objective for the day: use the R.Z.T. with our other techniques for dealing with polynomials to find roots, factor, solve problems, etc. I write this on a side board.

Stimulate Recall of Prior Learning II/Present the Content/Provide Learning Guidance I[[1]](#footnote-1) (2-7 minutes)

Class reviews battery of pre-assessment exercises (**4-8: odd exercises 1-9**), with me presenting answers at the board. Students are able to ask for full or partial working out of an exercise or two; if possible, I deflect this working-out to students themselves, calling one to present and explain at the board or among their own group. It is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Flipped structure means that main presentation of content occurs before class work time. It is essential, however, to assess student uptake of this by examining notes and work on pre-assessment exercises and asking assessing questions about these. In addition to completion of notes and pre-assessment exercises, it will be important to look for:

* Implementation of R.Z.T.
  + Correct identification of lead and constant terms.
  + Identification of all factors of each.
    - Both positive and negative.
  + Formation of set of quotients of these.
    - With factors of constant in numerator.
* Use of efficiency strategies with R.Z.T.
  + F.T.A. to limit number of candidates checked.
  + Factoring/checking depressed polynomials.

Assessing questions for student understanding include:

* How did you come up with these candidates?
* Why didn’t you check [non-candidate]?
* Could [irrational number] have been a root?
* How many of these did you check?
  + Why did you stop after that number?

Students are released to work with group support on in-class assignment as they and I are satisfied that they understand the presented content from pre-class session. Possibly, some groups or students will require remediation to reach the level required to do this work, particularly if students have failed to attend to their notes or video tutorials.

Provide Learning Guidance II/Elicit Performance (Practice) (30-35 minutes)

Students work with small group support on battery of exercises from our text (**4-8: odd-numbered exercises 11-37; even-numberd exercises 52-62**). As students work, cooperating teachers ask assessing questions and pose advancing prompts, especially noting any areas of general difficulty and specific student needs. Prompts fade as students progress through battery, increasing independence.

Provide Feedback (0-2 minutes – mainly in succeeding lesson & built into classwork)  
Main avenues for feedback are advancing prompts discussed above and self-marking built into succeeding lesson. Further, students have opportunity to correct self-marked errors to recover credit. In-lesson, group and individual are main units of feedback; rarely, consider breaking work time to address areas of general difficulty with the whole class.  
  
For this lesson, items to especially notice include:

* Over- or under-checking.
  + Checking after all zeros found.
  + Checking when easier to find zeros of depressed polynomial by other means.
  + Finding/checking only rational zeros, or giving up after exhausting the R.Z.T. set.
* Continued use of efficient vs. inefficient techniques.
  + Incorporation of new technique into overall strategy vs. abandonment of previous techniques.
* Continued finding vs. non-finding of non-real zeros.

Assess Performance (0 minutes)

Main avenue of formal evaluation is unit test, occurring after segment. Semi-formal assessment is obtained by work product and self-marks on classwork, obtained in succeeding lesson. Assessing prompts noted in above items provide further scope for informal assessment.

Enhance Retention and Transfer (2 minutes)

Much work toward this objective is done by revisiting content in succeeding lesson. To further it, toward end of classwork time, deliver short verbal “wrap” summarizing what we’ve done today:  
“Today, we added the last tool to our box to deal with polynomials. The R.Z.T. let us ‘work smarter,’ by saving us from checking every number under the sun.  
  
We’ve built a powerful set of tools to deal with polynomials: we can graph, factor, and use synthetic evaluation; we know exactly how many zeros to look for, and what candidates to check and avoid. When we use these right, they work together to make each better, like good tools should. Tomorrow, enjoy your day off and consider making some corrections. When we next meet, we’ll revisit what we did this week and maybe find that some problems look easier now.”

1. Flipped structure mingles these areas. [↑](#footnote-ref-1)