**Lesson Plan 2**

4-7: Roots and Zeros (The Fundamental Theorem of Algebra)

Purpose and Central Focus

This lesson is part of a learning segment on the roots of polynomials. Students will connect ideas from previous sections of the unit and segment,formalizing the relationship among roots of a polynomial equation, zeros of a polynomial function, factors of a polynomial, and *x*-intercepts of a polynomial function’s graph – namely, that finding any is equivalent to finding each. This synthesis leads in to a central idea for the course: the Fundamental Theorem of Algebra (F.T.A.), namely, that each polynomial has exactly as many zeros as its degree, with the complication that some zeros may be non-real. The new-found ability to bound the number of zeros will be immediately put to use in factoring and zero-finding, coupled with the special techniques of Descartes’ Rule of Signs and the Complex Conjugates Theorem (C.C.T.).

As its name implies, the F.T.A. is absolutely central to this branch of mathematics – its connections to other areas would be difficult to over-state. As mentioned above, we mainly use its implication here to aid our root-finding, but it also connects our analysis of real polynomials to complex numbers and properties of polynomials used as models.

Illinois Learning Standards/Common Core Standards/Content Area Standards

* HSN.CN.C.9 – Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
* HSA.APR.B.3 – Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Objectives

1. Given a polynomial *P* (C), the learner (A) will use the Fundamental Theorem of Algebra to state how many (complex) zeros it has (B) in 100% of cases (D).
2. Given a polynomial *P* with a non-real zero *a* + *bi* (with *a* and *b* real) (C), the learner (A) will use the Complex Conjugates Theorem to find another non-real zero, *a* – *bi* (B), in 90% of cases (D).

Duration

47 minutes.

Materials and Equipment

* Student note packet.
* Video tutorials for direct presentation and example exercises. (Including supplemental presentations on complex number system and Descartes Rule of Signs.)
  + Website to disseminate of these.
* Student and teacher textbooks (McGraw Hill *Integrated Math 3*).
  + Student answer sheets for marking previous work (**4-6: odd exercises 9-27, 45-53, & 59**).
  + Worked-out pre-assessment exercises (**4-7: odd exercises 1-3 & 9-15**).
* Graphing calculators.

Stimulate Recall of Prior Learning I (3 minutes)

Students mark their own results on classwork from previous lesson (**4-6: odd exercises 9-27, 45-53, & 59**). Students may ask for review of certain exercises, but it is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Gaining Attention/Introduction/Anticipatory Set (3 minutes)

Class uses flipped structure, so students come to class having viewed video tutorials and done pre-assessment exercises. At start of in-class work time, as I check their note work, students are challenged to come up with a quadratic polynomial that has “nice” (real, integer) coefficients but “not nice” (complex) zeros. I ask a couple students to share theirs.

Inform Learners of Objectives (3 minutes)

Citing student example polynomials, I notice that each has exactly 2 zeros, and their pairs of zeros are complex conjugates – each zero has a real and imaginary part, with the imaginary parts having opposite signs. I then ask students to explain how this is accordance with the Fundamental Theorem of Algebra and Complex Conjugates Theorem. Then I state our objective for the day: use the F.T.A. and C.C.T. with the zero-finding techniques we already know to find zeros, factor, and solve. I write this on a side board.

Stimulate Recall of Prior Learning II/Present the Content/Provide Learning Guidance I[[1]](#footnote-1) (2-7 minutes)

Class reviews battery of pre-assessment exercises (**4-7: odd exercises 1-3 & 9-15**), with me presenting answers at the board. Students are able to ask for full or partial working out of an exercise or two; if possible, I deflect this working-out to students themselves, calling one to present and explain at the board or among their own group. It is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Flipped structure means that main presentation of content occurs before class work time. It is essential, however, to assess student uptake of this by examining notes and work on pre-assessment exercises and asking assessing questions about these. In addition to completion of notes and pre-assessment exercises, it will be important to look for:

* Consistently correct statement of number of zeros.
* Attempts at finding non-real zeros.
  + The extent to which they are successful.
* Finding of roots in or not in accord with the C.C.T.
* Continued adoption of factoring of depressed polynomial technique from previous lesson.

Assessing questions for student understanding include:

* How do we know that this equation has [n] roots?
* How did you find this root?
  + Was that the only way?
* Do you notice anything interesting about these roots?
  + (Especially if roots are complex conjugates.)
* Can you say anything about when an equation will have a non-real root?

Students are released to work with group support on in-class assignment as they and I are satisfied that they understand the presented content from pre-class session. Possibly, some groups or students will require remediation to reach the level required to do this work, particularly if students have failed to attend to their notes or video tutorials.

Provide Learning Guidance II/Elicit Performance (Practice) (30-35 minutes)

Students work with small group support on battery of exercises from our text (**4-7: odd-numbered exercises 17-25, 33-43, 47-49, & 61-71**). As students work, cooperating teachers ask assessing questions and pose advancing prompts, especially noting any areas of general difficulty and specific student needs. Prompts fade as students progress through battery, increasing independence.

Provide Feedback (0-2 minutes – mainly in succeeding lesson & built into classwork)  
Main avenues for feedback are advancing prompts discussed above and self-marking built into succeeding lesson. Further, students have opportunity to correct self-marked errors to recover credit. In-lesson, group and individual are main units of feedback; rarely, consider breaking work time to address areas of general difficulty with the whole class.  
  
For this lesson, items to especially notice include:

* Continuing to look for zeros/roots/factors after all have been found vs. using the F.T.A. to determine the number of these from the outset and using that knowledge.
  + This is the essential new learning for this lesson, so notice and correct this quickly.
* Use of efficient (use of depressed polynomial; synthetic evaluation; factoring where available) vs. inefficient (guess and check, especially with direct evaluation) techniques.
* Finding vs. non-finding of non-real zeros.

Assess Performance (0 minutes)

Main avenue of formal evaluation is unit test, occurring after segment. Semi-formal assessment is obtained by work product and self-marks on classwork, obtained in succeeding lesson. Assessing prompts noted in above items provide further scope for informal assessment.

Enhance Retention and Transfer (1 minute)

Much work toward this objective is done by revisiting content in succeeding lesson. To further it, toward end of classwork time, deliver short verbal “wrap” summarizing what we’ve done today:  
“Today, we used some very powerful new tools, the Fundamental Theorem of Algebra and the Complex Conjugates Theorem, to attack polynomial equations. Did you notice how these worked well with the techniques we already know, like factoring and synthetic substitution? In today’s video lesson, we’ll add the final piece to our polynomial toolbox.”

1. Flipped structure mingles these areas. [↑](#footnote-ref-1)