**Lesson Plan 1**

4-6: Remainder and Factor Theorems

Purpose and Central Focus

This lesson is part of a learning segment on the roots of polynomials. Students will learn and apply the idea that a polynomial can be evaluated using its remainder when divided by another polynomial, which allows finding values and zeros in a new, often more efficient way. This in turn underpins development increasingly efficient suite of techniques for analyzing polynomials, using techniques for division, factoring, and evaluation developed in previous lessons and laying the groundwork for further refinement in succeeding lessons.

Understanding polynomials is essential to algebra and its applications. For instance, mathematical models of phenomena often take the form of polynomials. Consequently, a learner must understand a great deal about polynomials to interact with, use, or create such models, and use them in turn to learn about the world.

Illinois Learning Standards/Common Core Standards/Content Area Standards

* HSA.APR.B.2 – Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).
* HSF.IF.C.7.C – Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Objectives

1. Given a polynomial *P* and value *c* (C), the learner (A) will correctly evaluate *P*(*c*) using synthetic substitution and the Remainder Theorem (B) in 90% of cases (D).
2. Given a polynomial *P* and a linear *x – c* (C), the learner (A) will determine whether *x – c* is a factor of *P* AND give the depressed polynomial *P*(*x*) *÷* (*x – c*) in 80% of cases (D).

Duration

47 minutes.

Materials and Equipment

* Student note packet.
* Video tutorials for direct presentation and example problems.
  + Website for dissemination of these.
* Student and teacher textbooks (McGraw Hill *Integrated Math 3*).
  + Student answer sheets for marking previous work (**“Solving Polynomial Equations Skills Practice” worksheet**).
  + Worked-out pre-assessment exercises (**4-6: odd-numbered exercises 1-7**).
* Graphing calculators.

Stimulate Recall of Prior Learning I (3 minutes)

Students mark their own results on classwork from previous lesson (**“Solving Polynomial Equations Skills Practice” worksheet**.) Students may ask for review of certain exercises, but it is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Gaining Attention/Introduction/Anticipatory Set (5 minutes)

Class uses flipped structure, so students come to class having viewed video tutorials and done example problems on lesson subject. At start of in-class work time, as I check their note work, students are challenged to come up with an answer to: “What’s the point of this? Can’t we just evaluate polynomials the old way?” Some students are asked to share their answers.

Inform Learners of Objectives (2 minutes)

Using student responses to the uses of the Factor and Remainder Theorems and their accompanying technique of Synthetic Substitution, I directly approach the objective that we use this new technique to evaluate polynomials and find their roots and factors. This objective is refreshed verbally and displayed throughout in-class work time.

Stimulate Recall of Prior Learning II/Present the Content/Provide Learning Guidance I[[1]](#footnote-1) (2-7 minutes)

Class reviews battery of pre-assessment problems (**4-6: odd-numbered exercises 1-7**) with me presenting answers at the board. Students are able to ask for full working out of a problem or two; if possible, I deflect this working-out to students themselves, calling one to present and explain at the board or among their own group. It is important to limit the duration of this section of the lesson to maintain integrity of structure. It may be necessary to leave some questions pending for classwork section.

Flipped structure means that main presentation of content occurs before class work time. It is essential, however, to assess student uptake of this by examining notes and work on pre-assessment problems and asking assessing questions about these. In addition to completion of notes and pre-assessment problems, it will be important to look for proper technique, including use of synthetic as opposed to direct evaluation and use of depressed polynomials after location of a root allows dividing out a factor. Good assessing questions include:

* What would you get if you evaluate this directly/just “plug in” [number] into this polynomial?
* ­­­­­­­How can I use this zero to factor the polynomial?
* Why did you use the [full/depressed] polynomial after you found this root? Why does that work?­

Students are released to work with group support on in-class assignment as they and I are satisfied that they understand the presented content from pre-class session. Possibly, some groups or students will require remediation to reach the level required to do work, particularly if students have failed to attend to their notes or video tutorials.

Provide Learning Guidance II/Elicit Performance (Practice) (29-34 minutes)

Students work with small group support on battery of problems from our text (**4-6: odd-numbered problems 9-27, 45-53, & 59**). As students work, cooperating teachers ask assessing questions and pose advancing prompts, especially noting any areas of general difficulty and specific student needs. Prompts fade as students progress through battery, increasing independence.

Provide Feedback (0-2 minutes – mainly in succeeding lesson & built into classwork)  
Main avenues for feedback are advancing prompts discussed above and self-marking built into succeeding lesson. Further, students have opportunity to correct self-marked errors to recover credit. In-lesson, group and individual are main units of feedback; rarely, consider breaking work time to address areas of general difficulty with the whole class.  
  
For this lesson, items to especially notice include:

* Use of direct evaluation vs. synthetic evaluation.
  + Sticking to direct evaluation may indicate non-understanding of Factor Theorem.
* Use of full polynomial vs. depressed polynomial after initial factoring.
* Proper implementation of division algorithms in synthetic evaluation.
  + Student errors in algorithms may confound results. Use problems where factor is given to be checked for assessment.
* Recognition of conditions for division to produce zero/factor.
* Continuing to look for zeros after all have been found.
* Exclusive use of evaluation rather than direct factoring methods.
  + Especially on tractable depressed polynomials. Therefore, likely to be closely related to item 2.

Assess Performance (0 minutes)

Main avenue of formal evaluation is unit test, occurring after segment. Semi-formal assessment is obtained by work product and self-marks on classwork, obtained in succeeding lesson. Assessing prompts noted in above items provide large scope for informal assessment.

Enhance Retention and Transfer (1 minute)

Much work toward this objective is done by revisiting content in succeeding lesson. To further it, toward end of classwork time, deliver short verbal “wrap” summarizing what we’ve done today:  
“Today, we’ve used the Remainder Theorem to build a new tool: synthetic substitution. This let us evaluate polynomials in a new way, by finding zeros efficiently. We then used that new ability to factor polynomials. In today’s video lesson, we’ll build yet another tool to deal with polynomials that will work very well with synthetic substitution, making it, and you, even more powerful.”

1. Flipped structure mingles these areas. [↑](#footnote-ref-1)