Michael Angelo Sorice

Teaching Secondary Mathematics

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Thinking Through My Lesson Protocol – Make a Saving Throw Against Losing Your Seat

1. (a) Students will recover the binomial distribution by modeling a simple random process (rolls of several 10-sided dice.) They will then discuss the meaning of this distribution in other contexts and apply it to model an analogous quantity (number of no-shows for an airplane) as a binomial random variable. They will then use that variable to estimate various quantities of interest as expectations of derived variables in a few different ways. Finally, they will consider limitations to the model and how parameters that enter the expectation change decisions made by the model.

   (b) Students should have a reasonable degree of familiarity with random variables in general, particularly the concept of expectation. Students will need to be proficient users of probability, particularly the method of complements. Finally, students will need a fairly high degree of familiarity with the binomial distribution – c.d.f and p.d.f. – including its meaning, its functional form and the meanings of its parts, and the practical ability to compute values of forms for fairly large parameters using technology or other means.

   (c) Students will extend a known model by analogy and generalization. In doing so, students will learn to form derived random variables modeling quantities of interest and compute their expectations to make predictions, eventually realizing the effect of the parameters of derived variables in their expectations in terms of the expectations of parent variables.

(d) As a whole, this lesson best engages with `MP4: Model with mathematics`. Students will build models of quantities of interest as random variables, by analogy to a known example and then derivation. These models will then be used to make predications and draw conclusions about modeled situations. Students will further engage with the validity of models by argument about what makes a model work and its limitations.

This lesson also engages with `MP7: Make use of structure`. Students will use structural analogies to extend a known model to one about no-shows for a plane. Computations with derived random variables from the latter exhibit important patterns that lead to adoption of more efficient methods with reflection, as well as the central realization: that the expectation of a variable derived by linear transform is the same linear transform of the expectation of the parent variable.

2. (a) The main tasks of this lesson are to re-establish a known model and then extend it by analogy to a variety of unknown situations, and then to use these derived models to make predictions and estimates by taking expectations.

(b) Students are encouraged – required, at points – to pursue a variety of paths to solution, which include different methods of computation, symbolic certainly and possibly graphical. That being said, the lesson does converge on using random variables and c.d.f. for efficient computation. This is by design as these methods are more efficient in the regimes under consideration.

(c) I would expect that students may have shaky concepts of random variables and expectations – in particular, unexperienced practitioners tend to conflate either or both of these with mean. While dealing with these is part of the essence of the lesson, it is not meant as an outright introduction to these concepts and students without at least some extent of familiarity with them will find this lesson daunting.

I also expect that at least some students will persist in familiar but inefficient methods of computation – e.g. summing 7 terms of a p.d.f. when a compliment method allows summing only 4 or a c.d.f. method, 2. As was the case above, this difficulty is dealt with to a degree in the activity by design, but failure to consider efficiency and evolve past familiar methods will greatly hinder progress after a certain point.

3. The lesson feels well-designed in this regard (if I do say so myself,) as it leads students into the unfamiliar from the very familiar reasonably gradually. The prepared teacher will take the usual precautions in this regard, mainly limiting the tendency to say much and instead to ask questions and let the lesson do its job.

4. (a) Instructions for students are clearly delineated in this regard. By design, students work in pairs or small groups in the central sections of the lesson, a structure which has a fair degree of inherent responsibility-promotion.

   (b) Students will need tools allowing computation of binomial p.d.f. and c.d.f., probably a calculator with statistics package or similar technology. Tabular or graphical forms of the binomial distributions for various parameters may substitute for or supplement technology.

(c) Beyond the misconceptions listed at 2c, it is possible that some students may have the major conclusion, which is rather intuitive in some ways, as a heuristic without understanding it in its full rigor. It will be necessary to challenge these students with a number of advancing questions. The design of the activity will naturally impel other methods of calculation that would lead to a more rigorous foundation, but it is possible that students who feel they "already know" the conclusion may resent these.

5. (a) Class structure can be useful in this regard, the small-group paradigm tending to promote engagement. As always, careful observation of characteristic and assessing questions (such as those at 5c and 6, below) are crucial in this task as well.

(b) To help a student to begin, I would consider simpler, perhaps even more familiar starting examples, such as a single roll of a 6-sided die or flip of a coin (Bernoulli processes,) then a fixed, finite number of such. Similar simplifications of model are readily available until at least the middle, `Sky Pilot` section.

(c) From students successfully at work on this task, I expect to hear exchange of mathematical ideas with partners including precise use of mathematical terms like *random variable*, *expectation*, and *c.d.f.* I expect to hear cogent arguments when responses to the expect portion of question 1 (an average, not what we "expect" to get in the common meaning of that word) and 4 (the binomial used for the dice is good because there are a lot of people who show up or not for a variety of random reasons, for example.) I expect to see multiple computation methods in the first, `Tumbling Dice` section, followed by careful selection of more efficient methods in subsequent sections.

6. (a) To support students working well, I would pose open prompts like: "Describe what you are thinking about this." I may also consider extensions, such as "Does the airline always make that much money? How much can their take vary from launch to launch?" This is a good time to remind ourselves once again not to intervene too much when students are working well, which is a known tendency in my own teaching.

   (b) Questions like "Why are you doing that?" or "What do you expect to find?" are good, open questions that are almost always useful in this regard. "Which way seemed easier to compute this?" is an important question of this type that may be asked at several points throughout the lesson.

   (c) Useful advancing questions might go along the lines of: "Is there more than one way to write that quantity?" or "Did you try writing out what you're going for as a sum first?" "How are [item] and [item] related?" may serve this purpose at several later points.

7. (a) I will have a good idea that the goals of this lesson are being met if students are able to answer question 6 with a fair degree of fluency, as this practically requires adoption of an efficient method of computation by comparison of the various methods. Clearly, successful completion of the first part of the middle section

capstone question, 11, is a key indicator that the goals have been met.

(b) Questions 4 and especially 11 are excellent probes of `MP7`. Questions 5 and especially 13 give strong evidence of the degree of engagement of `MP4`.

8. (a) I will feel safe to begin the share/summarize phase when all or almost all groups have a plausible answer for the very first part of question 11. Comparison of results of these across groups, as long as one is successful, will likely inform all groups of the efficient technique and underlying principle – commutativity of expectations with linear transformations of variables – here.

   (b) Student work will be sequenced based on how complete an answer to question 11 is obtained. Students will briefly present their own results to problems 10 and 11 in front of the class.

9. Some "inverse problem" extensions are readily available, such as: "How many dice must be rolled to be p% sure of less than n 1s?" (A form of this problem was in an earlier version of the lesson.) I would further extend this lesson by directing students to consider variances and other higher expectations with questions like "Do 149 people always show up? How many do?" I would also explicitly allow the other parameters, particularly no-show probability and sales rate, to vary in problem 14, creating a much more difficult quasi-optimization problem – for example "How low does the no-show rate have to be to make it never profitable to oversell?"