Michael Angelo Sorice

Teaching Secondary Mathematics

Spring 2019

February 27, 2019

Mathematics Lesson Plan – Make a Saving Throw Against Losing Your Seat

Mathematical Goals

- After recovering the binomial distribution for a fixed number of trials of given fixed probability and discussing the meaning of this as the probability of a number of successes given the parameters, students will use it as a model of various events.
- Students will perform various computations with a random variable, making use of structure and reflection to select efficient methods.
- Students will use a parent random variable and derived variables to model situations analogous to familiar ones and then situations related to those, computing parameters of interest as expectations by direct and derived methods.

Standards This lesson is best aligned with the following Common Core State Standards for Mathematics Content:

- `HSS.MD.A.2: Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.`
- `HSS.MD.A.3: Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.`

- `HSS.MD.B.7: Analyze decisions and strategies using probability concepts.`

Students will make most extensive use of Common Core State Standards for Mathematical Practice:

- `MP4: Model with mathematics.`
- `MP7: Look for and make use of structure.`

Students' Needs

- Good basis with random variables and expectations, including at least linear transformations thereof.
- Fair understanding of probability, especially compliment method.
- Moderate familiarity with mathematical expectation, but without detailed knowledge of how it commutes with transformations of variables.
- High degree of familiarity with binomial distribution, both p.d.f. and c.d.f., and ability to compute same using technology.

Lesson Details

Launch I will introduce this activity by having students work in pairs or small groups on a series of problems in the "Before"/`Tumbling Dice` phase, re-introducing the binomial distribution and the notion of mathematical expectation, with particular attention to the question "How can we expect something that doesn't happen most of the time?" (`MP4`.) Further problems in this section are applications of the same variable in which computation of expectations and probabilities derived from that variable are considered in several ways, with structural differences driving greater efficiency in some methods (`MP7`, `MP1`.) These can be worked more or less autonomously, based on assessment of the groups' proficiency.

Explore As students begin the "During"/`Sky Pilot` phase, they will continue in small groups to first make and then share arguments why the same binomial variable that describes dice can also describe no-shows to airlines (`MP4`,) with selected arguments shared and debated by the class. They will then use this model to make various predications by calculating expectations (`MP4`,) with structure and reflection again informing the methods of computation, albeit to a lesser degree (`MP7`.) In the penultimate part of this phase, students explicitly cast new random variables representing related quantities of interest (`MP4`, `MP7`.) Finally, this phase for students ends with a capstone question where the parameters of interest to the modeled situation are obtained as expectations of the new variables (`MP4`,) in multiple ways, with structure playing a key role in relating their equivalence and allowing selection for efficiency (`MP7`.) The completion of the first part of this capstone question is a signal to begin the next phase.

The teacher's main role during this work is to observe and advance students without undermining their productive struggle (`MP1`.) It will be crucial to assess progress by probing completion and cogency in key questions, 2, 5, and 11. Sequencing and orchestrating sharing of results of question 5 is another key job.

The small group work structure tends to amplify student accountability, though teachers much remain vigilant to re-focus and/or gently advance groups who are not working, as non-accountability and contagious sloth or lack of progress are known possible issues with this structure. Sharing of individual and collective reasoning and the expectation of probing for sense-making by teacher and peers are always norms of a good mathematics classroom and will pay important dividends here.

Share/Summarize Groups will be called to present their results to the capstone problem 11 for comparison, sequenced by degree of completion, with reasons, results, and methods the subject of debate by members of the class. All members of the small groups will speak for their group's results, so that everyone is accountable for participation. The use of productive talk moves can plug otherwise unengaged students into this debate.

Following this group portion, an individual after portion, `Leaving on a Jet Plane`, will allow assessment of individual learner progress and understanding at a rather fine level. This section presents extensions like the consideration of different, non-binomial models and the variation of parameters considered fixed and their possible retrograde effect on the situation modeled, given the implications of the model for that situation. Further extensions are readily available, such as consideration of the possible and plausible ranges of the derived variables, inverse problems, and higher moment expectations (starting with variance.)

Assessment Throughout this lesson, students will be challenged with a number of assessing prompts to probe their achievement of objectives and participation in practices. Questions to be posed during sharing, such as "Which way of computing probability works best for large numbers of trials? Why?" join with the implication of the results of others (ability to quickly compute approximate inverse of c.d.f. at 4 and 7, indicating computational fluency derived from structural considerations; cogency of responses to questions 2 and 5, indicating thought about modeling) to produce a reasonable picture of the extent of learning. Super-added to these are the result of oral assessing questions posed by the teacher, such as "Will that always work?" and "How did you know to combine expectations in that fashion?"

Important indicators of understanding include: precise use of terms and symbols in discussion and responses to probes, especially in section on casting random variables; explicit consideration of the difficulty of inefficient methods; cogent arguments for and against models at 1-2, 4, and especially 13.

Accommodations Accommodations for advanced students are partially addressed in the extensions above. A particular class of advanced student for whom this task is not appropriate without accommodation is one who already reasonably thoroughly understands the properties of the expectation of a function of a random variables in terms of the expectations of the parent variables. The "higher-order moment" and "inverse problem" extensions will be useful for such students. So may a slight re-design of the problem so that at least some transformed variables are non-linear, e.g. "What if a new law means that the airline has to pay the n^{th} 'bumped' passenger $n \times \$1000$ instead of $1,000?"

Students requiring additional support may be given a scaffold in the form of a review of simple trials, which follow Bernoulli and then limited binomial distributions. It is probably not appropriate to show such students efficient methods from the outset, rather than allowing them to be derived by comparison, as this comparison by structure is an essential part of the lesson; however, the number of different options for method may be reduced to just 2, say. Finally, easier choices of parameters may allow students to focus more on underlying mathematics without completely removing computational difficulties – for example, a small airline may use only 15-seat planes.