Make a Saving Throw Against Losing Your Seat

Adapted from Mathalicious "Bumpy Ride"
ASSIGNMENT FOR SMALL GROUPS – <u>BE PREPARED TO SHARE</u>!

Section 1: Tumbling Dice

Chancy situations are simulated by rolls of dice. For example, Mr. Sorice used to play a game where some 10-sided dice were rolled, and bad things happened if too many of them came up 1.

1. If you roll some fair 10-sided dice, what fraction of them would you expect to come up 1?

2. What do you mean by *expect* in #1?

3. How we can use a *binomial random variable* to answer #1 & #2?

4. 10 fair 10-sided dice are rolled. Compute the probability that 3 or more 1s show up in three ways:
    a. Sum the *binomial p.d.f.* directly.

    b. Use the *bionomial p.d.f.* and *method of complements.*

    c. Use the *binomial c.d.f.*

Section 2: Sky Pilot

Did you know that about 10% of ticket-holders no-show for a typical flight? It's true!
This can make flying more complicated for us responsible people. Let's see how:

5. Sketch a couple good arguments why the binomial variable we came up with for the number of 1s on a roll of several 10-sided dice might be a good one to model how many people won't show up for a given flight.

6. Let's say a flight seats 150 passengers, but the airline sold 167 tickets (can you see why they'd do that?) Using a binomial model, how likely is the plane to be full, i.e. to have at least as many people show up as there are seats?

7. When someone shows up for a flight, but it's already full, that's called being "bumped." How many people get "bumped" on average from the flight in #6? Is it the same as the number of tickets sold minus the number of seats? Why?

Panther Airlines flies exclusively 150-seat planes and sells 167 tickets for each flight. (Demand to fly out of Coles Co. Airport is very high, so all tickets sell.) PanAir's flights leave full a good portion of the time, but people are getting bumped pretty regularly. People have a right to compensation when they get bumped!

Let's say that underlying economics and law dictate that PanAir makes <u>$10,020</u> in fixed net revenue each time it flies – from non-refundable fees, etc. It also makes <u>$300</u> in variable revenue per person for each passenger who actually flies. However, it also has to pay out <u>$1,000</u> in compensation to each person it has to bump.

We can use *random variables* and *expectations* to see where the money's going. For convenience, let's use the name S for the random variable: <u>the number of ticket holders who show up for a given flight</u>. (In #4, we argued that S is binomial.)

8. Write a new variable, F, in terms of S, that gives the number of passengers who actually fly on a PanAir flight. (Be careful that F accurately reflects what happens to the revenue when more than 150 people show up!)

    o   Then write another new variable, R, that gives the variable revenue if F people are flying.

    o   Finally, re-write R in terms of S.

9. Write a new variable, B, in terms of S, that gives the number of people who get bumped from a flight if S people show up.

    o   Then write a second new variable, C, for what the airline has to pay in compensation when B people get bumped.

    o   Finally, re-write C in terms of S.

10. How much dough does PanAir make on average each time it flies? Let's write the answer symbolically in a few ways:

    a. The expectation of a new random variable stated in terms of R and C.

    b. The expectation of a random variable in terms of S.

    c. If possible, directly in terms of the binomial c.d.f. or p.d.f.

11. Evaluate as many of your expressions from #10 as you can to produce a final number.

Section 3: Leaving on a Jet Plane

Statistics and probability have given us some idea why airlines operate the way they do. Let's look at a couple of these issues in a little more depth.

WRAP-UP QUESTIONS
PLEASE DO THESE ON YOUR OWN AND TURN THEM IN ONCE YOU'RE FINISHED, OR FOR HOMEWORK

12. One form of PanAir's average profit per flight, $E(P)$, looks like this:

    $E(P) = \$10{,}020 + \$300\ E(F) - \$1{,}000\ E(B)$,

    where E is the expectation, F is the number of passengers who fly on each PanAir flight and B is the number of unhappy customers who got bumped. Do expectations always work out in this nice way? What are the limits on using them like this?

13. Can you think of any reasons that a random binomial model might not be a good model for no-shows to an airline? Name at least a few important limitations to this model. Can you think of a better one?

14. Make two concise arguments about how changes in the three rates (+\$10,200 per flight; +\$300 per seated passenger, -\$1,000 per bumped passenger) that dictate PanAir's profit per launch might result in changes in its policies.