Mike Sorice — College Geometry II
Spring 2019 — Homework -6

1   1)

Proposition. *Given AB in $\mathbb{N}^2$ with perpendicular bisector $\overleftrightarrow{AB^{\perp}}$, $P \in \overleftrightarrow{AB^{\perp}} \Rightarrow AP \cong BP$.*

Proof. Let $P \in \overleftrightarrow{AB^{\perp}}$ and let M be the midpoint of AB so that $\overleftrightarrow{AB^{\perp}} \cap AB = \{M\}$.
Connect P to A and to B.
$\triangle AMP \cong \triangle BMP$ by side-angle-side congruence ($AM \cong BM$ by definition of midpoint; $\angle AMP \cong \angle BMP$ by Euclid's IVth postulate, since both are right angles by definition of $\overleftrightarrow{AB^{\perp}}$; MP is a common side.)
$\therefore AP \cong BP$ as corresponding parts of congruent triangles. $\square$

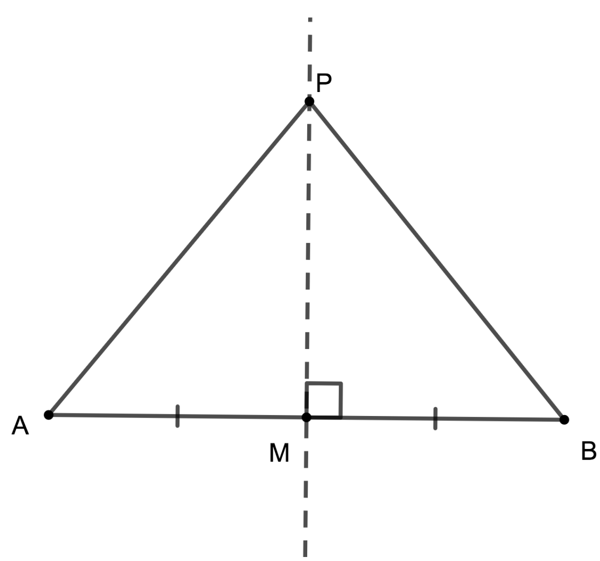


Figure 1: Point on perpendicular bisector is equidistant from endpoints.

2)

Proposition. *Given AB in $\mathbb{N}^2$ with $AP \cong BP$, $P \in \overleftrightarrow{AB^{\perp}}$.*

Proof. Either $P \in AB$ or $P \notin AB$.

Case 1 ($P \in AB$): Then $P = M$ by definition of midpoint, and $M \in \overleftrightarrow{AB^{\perp}}$ by definition of $\overleftrightarrow{AB^{\perp}}$. //

Case 2 ($P \notin AB$): Then $P \neq M$, so connect P to A, to B, and to M.

$\triangle AMP \cong \triangle BMP$ by side-side-side congruence ($AP \cong BP$ by hypothesis, MP is a common side, $AM \cong BM$ by definition of midpoint.)

$\therefore \angle AMP \cong \angle BMP$ by congruent triangles.

$\angle AMP$ also supplements $\angle BMP$, so both are therefore right triangles by definition.

$\therefore (MP \perp AB) \wedge (M \in MP) \Rightarrow \overleftrightarrow{MP} = \overleftrightarrow{AB}^{\perp}$.

$\square$

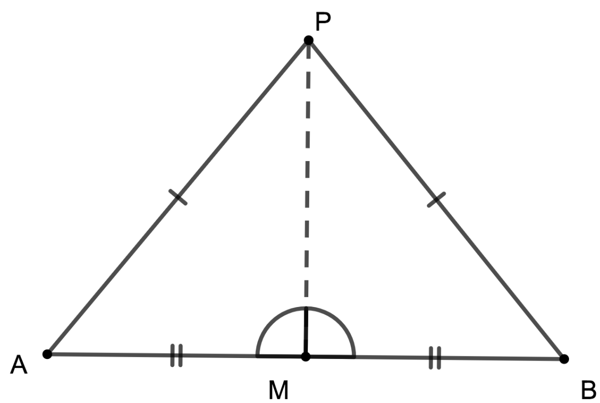


Figure 2: Point equidistant from endpoints is on perpendicular bisector.

(II)

Proposition. *Given AB in $\mathbb{N}^2$ with Q on B's side of $\overleftrightarrow{AB}^{\perp}$, $AQ > BQ$.*

Proof. Either $Q \in \overleftrightarrow{AB}$ or not.

Case 1 ($Q \in \overleftrightarrow{AB}$): $Q \neq A$, $Q \neq M$ and $\neg(A * Q * M)$ by hypothesis, so exactly one of: $A * M * Q * B$, $Q = B$, or $A * B * Q$.

If $A * M * Q * B$, then $|AQ| > |AM| = \frac{1}{2}|AB| = |BM| > |BQ| \Rightarrow AQ > BQ$. //

If $Q = B$ or $A * B * Q$, then obviously $AQ > BQ$.

Case 2 ($Q \notin \overleftrightarrow{AB}$): Since A is across $\overleftrightarrow{AB}^{\perp}$ from Q, there is some point P common where AQ meets $\overleftrightarrow{AB}^{\perp}$ by definition.

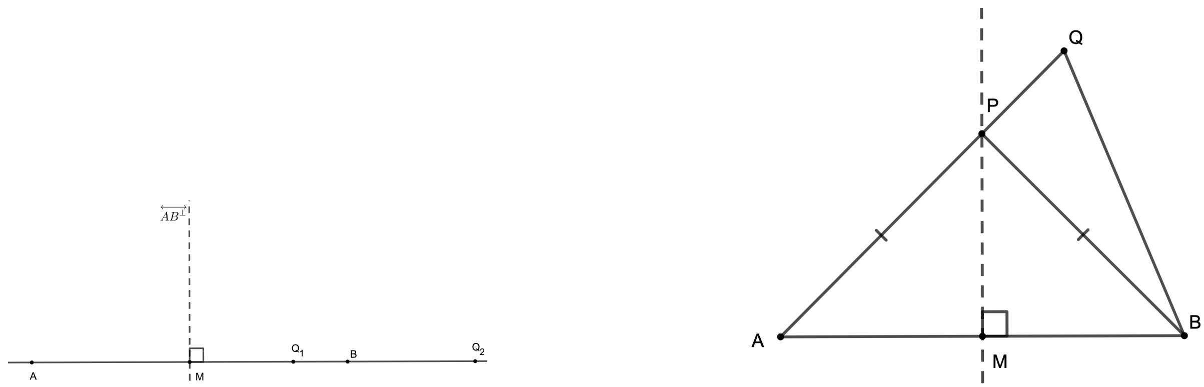
As $P \in \overleftrightarrow{AB}^{\perp}$, $AP \cong BP$ by previous result.

The triangle inequality on $\triangle BPQ$ yields: $|BP| + |PQ| > |BQ|$.

By definition, $AP + PQ = AQ \Rightarrow |AP| + |PQ| = |AQ|$.

Subtracting this from the above inequality yields: $|BP| - |AP| + |PQ| - |PQ| > |BQ| - |AQ|$, but $BP \cong AP$ by previous step, so $|BP| = |AP|$ and we have: $0 > |BQ| - |AQ| \Rightarrow AQ > BQ$.

□



(a) Whether on extension of segment...

(b) ...or not.

Figure 3: Point across perpendicular bisector from one endpoint is closer to other.