Mike Sorice | **College Geometry II**
Spring 2019 | **Homework -5**

Proposition. *Given* $\square ABCD$ *with* $AB \parallel CD$ *and* $AB \cong CD$ *in* $\mathbb{E}^2$, $\square ABCD$ *is a parallelogram.*

Proof by construction. $\angle BDC \cong \angle ABD$ as alternate interior angles of parallel lines by converse alternate interior angle theorem (in $\mathbb{E}^2$, parallel lines have congruent alternate interior angles.)

$\therefore \triangle ABD \cong \triangle CDB$ by side-angle-side congruence ($AB \cong CD$ by hypothesis, BD is a common side, $\angle ABD \cong \angle BDC$ by previous step.)

$\therefore \angle ADB \cong \angle CBD$ (corresponding parts of congruent triangles.)

$\angle ADB$ and $\angle CBD$ being alternate interior angles of AD and BC with transversal BD, $AD \parallel BC$ by alternate interior angle theorem (lines with congruent alternate interior angles are parallel.)

All pairs of opposite sides of $\square ABCD$ being parallel, $\square ABCD$ is a parallelogram by definition.

$\square$

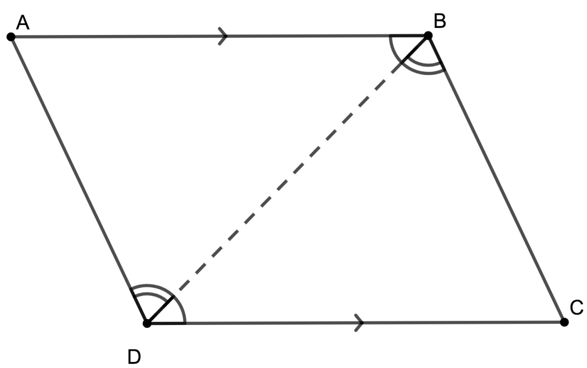

Figure 1: A quadrilateral with a pair of opposite sides that are congruent and parallel is a parallelogram in $\mathbb{E}^2$.