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College Geometry II
Homework -3

Greenberg's Ch. 4 Review Exercises

(1) False. All Euclidean triangles have the same defect.

(2) True. All right angles are congruent per our Proposition 3.23 (p. 92,) the proof of which does not rely on E-5.

(3) False. Theorem 4.5 shows that Hilbert's parallel postulate is equivalent to E-5, i.e. if we adopt either, we are in $\mathbb{E}^2$.

(4) False. We know only that no triangle has $\sum(\triangle) > 180°$ in $\mathbb{N}^2$. In $\mathbb{E}^2$, all triangles have $\sum(\triangle) = 180°$, whereas in $\mathbb{H}^2$, all have $\sum(\triangle) < 180°$. In neither geometry will we encounter a mix of these classes of triangles.

(5) False. The alternate interior angle theorem says the converse of this: if a transversal cuts congruent alternate interior angles, then the lines cut are parallel.

(6) False. Any four points such that no three are collinear will form a quadrilateral. That quadrilateral can be a rectangle only in $\mathbb{E}^2$, however.

(7) False. The Saccheri-Legendre theorem is a theorem of $\mathbb{N}^2$. $\mathbb{E}^2\triangle$s all having $\sum(\triangle) = 180°$ are compatible its requirement that $\sum(\triangle) \leq 180°$.

(8) True. Angles are non-opposite co-terminal rays. An "angle" measuring 180° by definition has a 0-measure supplement, i.e. it must be made of opposite rays.

(9) False. A ray is between two others if all are co-terminal and point of the ray except its vertex is interior to the angle of which the other two are sides.

(10) False. There is at least one line parallel to any given line through any point not on the given line in $\mathbb{N}^2$. In $\mathbb{H}^2$ there are at least 2 such lines, whereas in $\mathbb{E}^2$ there is exactly one.

(11) Somewhat surprisingly, this seems to be true. While the definition of "adjacent" seems obvious, I can't see it explicitly defined anywhere so far. We did define supplementary (p. 17) to mean what seems to be meant by adjacent.

(12) False. An exterior angle is the supplement to an angle of the triangle.

(13) True. The S.S.S. congruence criterion was proved as Proposition 3.22 (p. 92) in $\mathbb{N}^2$.

(14) False. A transversal may not cut congruent alternate interior angles, though the lines cut are parallel. Therefore, a transversal may be perpendicular to only one of a pair of parallel lines in $\mathbb{N}^2$.

(15) True. $\sum(\triangle) \leq 180° \Leftrightarrow \delta(\triangle) = 180° - \sum(\triangle) \geq 0°$.

(16) True. The A.S.A. congruence was proved in $\mathbb{N}^2$ as Proposition 3.17 (p. 90.)

(17) True. Theorem 4.7 requires us to lay off sides from a rectangle such that any other plane figure is interior to it. This requires Archimedes' axiom.

(18) False. This isn't even true in $\mathbb{E}^2$! Consider the perpendicular from either smaller angle of an obtuse $\triangle$.

(19) False. This is Hilbert's parallel postulate, which is only true in/defines $\mathbb{E}^2$.

(20) True. This was proved in $\mathbb{N}^2$ as Proposition 3.15 (a) (p. 88.)

(21) True. Theorem 4.1 is used to show that any other result than the conclusion of Theorem 4.2 would leave at least a pair of the sides of the triangle parallel, which is absurd.

(22) True. Precisely this is done on pp. 119-20.