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# College Geometry II
# Homework 2

Let $P$ be a point inside $\angle A = \angle \Sigma A \Omega$, with $A$ an ordinary point and $\Sigma$ and $\Omega$ ideal points. Let $\gamma = c(P, AP)$. Does $\gamma$ always meet the sides of $\angle A$...

**...in $\mathbb{E}^2$?**

> **Proposition.** *$\gamma$ always meets both sides of $\angle A$.*
>
> *Proof.* By Congruence Axiom 1, $\exists! S$ s.t. $S \in \overrightarrow{A\Sigma} \land AS \cong AP$.
>
> As $A$ and $AP$ are respectively the center and radius of $\gamma$ by construction, $AS \cong AP \Rightarrow S \in \gamma$ by definition of circle.
>
> $\therefore \overrightarrow{A\Sigma}$ meets $\gamma$ at $S$.
>
> Similarly, $\exists! W \in \overrightarrow{A\Omega} \cap \gamma$.
>
> $\square$

**...at some normal point in $\mathbb{H}^2$?** We made no use of Euclid's 5th postulate in the above, so $\mathbb{H}^2$ also obeys this proposition[1]. While it seems possible to construct counterexamples in models of $\mathbb{H}^2$ like this one:

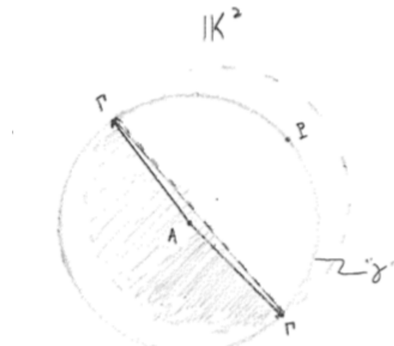


Figure 1: Seeming counterexample in $\mathbb{K}^2$. $\Gamma$ and $\Lambda$ are ideal points where "$\gamma$" meets the boundary. It seems that any angle whose sides are in the shaded area – on the side of $\Gamma\Lambda$ opposite $P$ – will therefore miss "$\gamma$"...

However, these rely on flawed reasoning. In this case, a flaw is that , while"$\gamma$" is a circle in the larger Euclidean plane in which $\mathbb{K}^2$ is embedded, some points of "$\gamma$"

[1]An interesting singular case may be if $P$ is an ideal point. In that case, $\gamma$ is the boundary of $\mathbb{K}^2$, so the sides of $\angle A$ meet $\gamma$ at $\Sigma$ and $\Omega$. Those points are outside $\mathbb{K}^2$, but then again, so is $\gamma$! I suspect the proper resolution to this is to say the proposition is vacuously true since there is no circle $c(A, AP)$ for $P$ an ideal point.

are further from $A$ in $\mathbb{K}^2$ than others. $AP < A\Lambda$, for instance[2]. Therefore, "$\gamma$" is by definition not a circle in $\mathbb{K}^2$.

[2] In fact $A\Lambda$ is infinite compared to $AP$!