Mike Sorice — College Geometry II

Spring 2019 — Region of Acute Triangle Vertices

Proposition 1. *Given AB, $\{C : \triangle ABC$ is acute$\}$ is the intersection of A's side of the perpendicular to AB through B, B's side of the perpendicular to AB through A, and the region strictly outside the circle whose diameter is AB.*

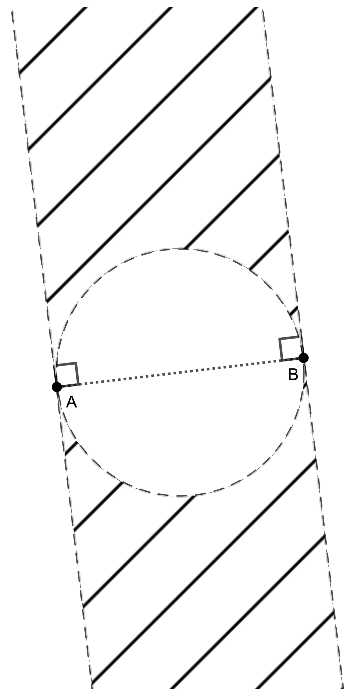

Figure 1: $\triangle ABC$ is acute if and only if C is in the hatched region.

Proof. Let p_A be the perpendicular to AB through A.

If C were on p_A, $\angle CAB = \angle A$ would be a right angle by definition of perpendicular, so $\triangle ABC$ would be a right triangle (and therefore not acute.)

Further, were C across p_A from B, $\angle A$ would be obtuse and $\triangle ABC$ not acute.

However, if C is on B's side of p_A, then $\angle A$ is acute.

$\therefore \angle A$ is acute if and only if C is on B's side of p_A. /

Letting p_B be the perpendicular to AB through B.

Then similarly, $\angle ABC = \angle B$ is acute if any only if C is on A's side of p_B. //

Let c be the circle whose diameter is AB.

By previous corollaries, $\angle C = \angle ACB$ would be obtuse were C interior to c, so $\triangle ABC$ would be obtuse were C there.

Similarly, $C \notin c$ because were it, $\angle C$ and $\triangle ABC$ would be right.

However, if C is exterior to c, then C is an acute angle.

$\therefore \angle C$ is acute if and only if C is outside c. /// $\square$